Test 3a, MAT157

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1. (10 pts) Suppose $f(x)$ and $g(x)$ are differentiable on $\mathbb{R}$ and $g'(x) = f'(x) + h(x)$, where $h(x)$ is a continuous function satisfying $h(x) > 0$ for $x > a$, $h(x) < 0$, for $x < a$. Suppose that $g(a) = f(a) + 3$. Show that $g(x) > f(x) + 3, \forall x \neq a$.

2. (10 pts) Suppose $0 < R < 1$. The horizontal planes $z = R$ and $z = -R$ intersect the unit sphere $x^2 + y^2 + z^2 = 1$ in circles containing points of the form (x, y, R) and $(x, y, -R)$. There are eight such points for which either $x = y$ or $x = -y$, and these eight points are the vertices of a rectangular solid. What is the maximum possible surface area of this solid, for $0 < R < 1$?

3. (10 pts) Let

$$f(x) = \begin{cases} 2, & \text{if } 0 \leq x < 1, \\ -3, & \text{if } x = 1, \\ 5, & \text{if } 1 < x < 3, \\ -1, & \text{if } 3 \leq x \leq 4, \end{cases}$$

Use an upper and lower sums argument to show that $f(x)$ is integrable on $[0, 4]$ and to evaluate $\int_0^4 f(x)\,dx$.

4. (10 pts) Working from the definition of $\sin x$ developed in class, prove that $\sin x > \frac{x}{2}$, whenever $0 < x < \frac{\pi}{2}$. You are allowed to assume we know the values of the trigonometric functions at the usual "special angles".

5. (10 pts) Find the area of the region enclosed between the graphs of $y = 7x^4 + 4x^3 - 6x^2 - 4x + 3$ and $y = 2x^4 + 4x^3 + 9x^2 + 6x + 3$.

6. (10 pts) Show that there is exactly one point P on the graph of $\arcsin x$, for $x \in [-1, 1]$, as defined in class, for which the tangent line to $y = \arcsin x$ at P is also a tangent line to $y = \sin x$, though possibly at some other point.