Problem 1. Prove there are no differentiable functions f and g satisfying $f(0) = g(0) = 0$ and

$$x = f(x)g(x) \tag{1}$$

Recall. A function f defined on an interval is convex if, for every a and b in the interval, the line segment connecting $\big(a, f(a)\big)$ and $\big(b, f(b)\big)$ lies above the graph of f.

Problem 2 (Spivak 11A-3)**.** Show that $f\colon [a, b] \to \mathbb{R}$ is convex if and only if for all $x,\ y \in [a, b]$, we have

$$f\big(tx + (1-t)y\big) < tf(x) + (1-t)f(y) \tag{2}$$

Problem 3 (Spivak 11A-10)**.**

a) Let $f\colon (a, b) \to \mathbb{R}$ be convex. Prove that f is continuous.

b) Find a function $g\colon [a, b] \to \mathbb{R}$ which is convex but *not* continuous. What fails in this case?