Problem 1 (Spivak 11-6)**.** Show if f is increasing on (a, b) and continuous on $[a, b]$, then f is increasing on all of $[a, b]$.

Problem 2 (Spivak 11-13)**.** Show the sum of a positive number and its reciprocal is at least two.

Problem 3. Suppose that

$$\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0 \tag{1}$$

Show that for some $x \in (0, 1)$,

$$a_0 + a_1 x + \cdots + a_n x^n = 0 \tag{2}$$

Problem 4. Let f and g be polynomials with $\deg f = m$ and $\deg g = n$.

1. Find an upper bound for the number of points c which satisfy $f(c) = g(c)$.
2. For each m and n, find an example where this upper bound is reached.

Problem 5. Suppose $f'(x) \geq M > 0$ for all $x \in [0, 1]$. Show there exists an interval $I = [a, b] \subseteq [0, 1]$ of length at least $\frac{1}{4}$ for which $|f| \geq \frac{M}{4}$.

Problem 6.

1. Prove that if $f'(a) > 0$ and f' is continuous at a, then f is increasing in some interval containing a.
2. Does the statement still hold if f' is not continuous? If not, can you find a counterexample?

Problem 7. Compute the following limits:

1. $\lim_{x \to 0} \frac{\sin x}{x}$
2. $\lim_{x \to 0} \frac{\cos x - 1}{x}$
3. $\lim_{x \to 0} \frac{e^x - 1}{x}$
4. $\lim_{x \to 0} \frac{x^2 - 1}{1 - \cos x}$

Problem 8. Find all functions f such that:

1. $f'(x) = \sin x$.

2. $f''(x) = x^3$.
3. $f^{(3)}(x) = x + x^2$.

Do any functions satisfy more than one of these conditions?

Problem 9. If $n \geq 1$, then for $x > -1$,

$$(1+x)^n \geq 1 + nx \tag{3}$$

Where equality holds if and only if $x = 0$.

Problem 10. Prove there are no differentiable functions f and g satisfying $f(0) = g(0) = 0$ and

$$x = f(x)g(x) \tag{4}$$