Problem 1. Given $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, let $f_n(x) = |x|^n$. For which values of n is f differentiable at 0?

Problem 2 (Inspired from Spivak 9-22)**.**

1. Suppose f is differentiable at x. Show that

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{h} \tag{1}$$

2. Does the converse hold? If the above limit exists, is f differentiable at x? Prove this is true or find a counterexample.

Problem 3 (Spivak 9-27)**.** Let $S_n(x) = x^n$ and $0 \leq k \leq n$. Prove that

$$S_n^{(k)}(x) = \frac{n!}{(n-k)!} x^{n-k} \tag{2}$$

Problem 4. A function $f\colon (a,b) \to \mathbb{R}$ is called Lipschitz if there exists a $k > 0$ such that for all x, $y \in [a,b]$,

$$\big|f(x) - f(y)\big| \leq k|x-y| \tag{3}$$

Prove that if f is differentiable on (a,b) and f' is bounded, then it is Lipschitz.