

Problem 1 (Gabriel's Horn). Gabriel the angel has an infinitely long horn (the front of which is depicted in figure 1) which looks like the revolution of the graph of $x \mapsto \frac{1}{x}$ on $[1, \infty)$ about the x -axis.

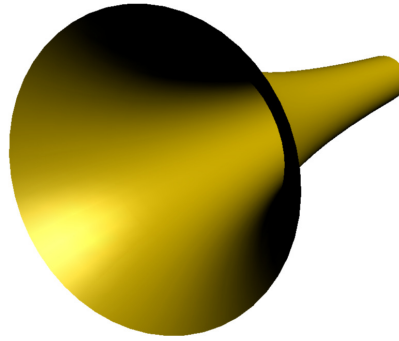


Figure 1: Gabriel's Horn

- a) Gabriel wants to repaint their horn. How much paint do you need to fill the horn?
- b) Either excited or disappointed at your previous answer, Gabriel offers to fill their horn with ice cream to either celebrate or console yourselves. How much ice cream fits in the horn?

Problem 2 (Spivak 19-9). Consider a simple closed curve in the plane. Construct a pyramid whose base is the region enclosed by the curve and whose height is h (see figure 2). What is the volume of this pyramid?

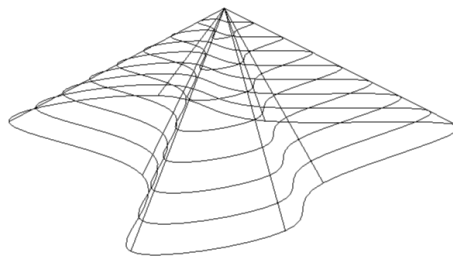


Figure 2: Pyramid

Problem 3. Consider cylinders C_x and C_y , each of radius 1 centred at the x - and y -axes respectively. Compute the volume of the intersection $C_x \cap C_y$. See figure 3.

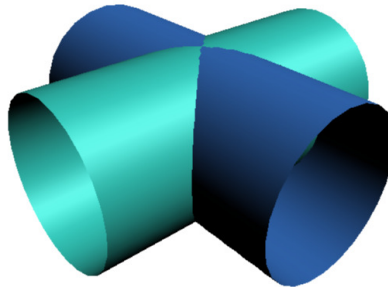


Figure 3: Cylinders

Problem 4 (Spivak 19-6). Compute the volume of revolution of $\cos x$ starting at 0 which lies above the xy -plane about the z -axis (figure 4) using:

1. the disc method (feel free to give up).
2. the shell method.

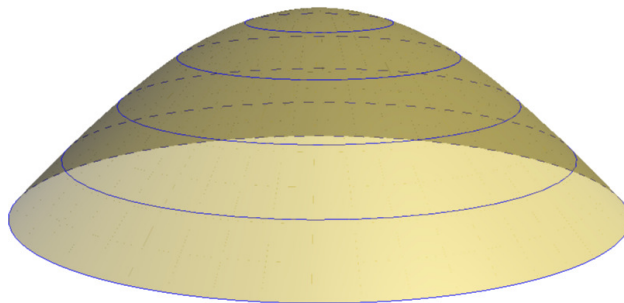


Figure 4: Revolution of cosine about the z -axis

Problem 5 (Potentially extremely hard). Compute the surface areas of the figures above if you haven't already.