Problem 1. For each $n \in \mathbb{N}$, compute the area bounded between the curves $y = x^n$ and $y = x^{n+1}$.

Problem 2 (Spivak 14-3.ii)**.** Show that the following expression is independent of x on $(0, \pi/2)$:

$$\int_{-\cos x}^{\sin x} \frac{dt}{\sqrt{1-t^2}} \tag{1}$$

Problem 3. Compute the following functions of t:

1. $$f(t) = \frac{d}{dt}\int_{t^2}^{1-t} x^2 \sin(x)\,dx \tag{2}$$

2. Let $h\colon \mathbb{R} \to \mathbb{R}$ be any integrable function.

$$g(t) = \frac{d}{dt}\int_{t}^{t+1} h(x)\,dx \tag{3}$$

3. Compute $F'(t)$ for

$$F(t) = \int_0^t t f(x)\,dx \tag{4}$$

4. Most generally, let a and b be differentiable functions, and let h let be integrable.

$$f(t) = \frac{d}{dt}\int_{a(t)}^{b(t)} h(x)\,dx \tag{5}$$

In the following problem, you may use this fact:

Fact 4. *For any natural number p, there exist numbers a_k such that*

$$\sum_{k=1}^{n} k^p = \frac{n^{p+1}}{p+1} + a_p n^p + a_{p-1}n^{p-1} + \cdots + a_1 n + a_0 \tag{6}$$

Problem 5 (Spivak 12-3)**.**

1. Show that for any $p \in \mathbb{N}$,

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^p}{n^{p+1}} = \frac{1}{p+1} \tag{7}$$

2. Without using FTC, prove that

$$\int_0^b x^p \, dx = \frac{b^{p+1}}{p+1} \tag{8}$$

Problem 6 (Spivak 14-9)**.** If f is continuous, show that

$$\int_0^x f(u)(x-u)\, du = \int_0^x \left(\int_0^u f(t)\, dt \right) du \tag{9}$$

Hint: Use (3) from problem 3.